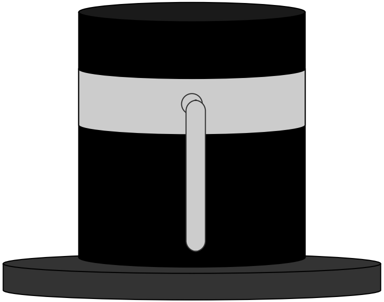
Ensuring Cyber-Physical System Stability in the Presence of Deadline Misses

Martina Maggio
Saarland University & Lund University

Modelling the Physical Phenomena[1]

Physical Quantities

- Pendulum arm length l: 0.075 $[m]$
- Base radius r: 0.043 $[m]$
- Base moment of inertia J: 0.000125 $[kg\ m^2]$
- Pendulum mass m: 0.00544 $[kg]$
- Gravity g: 9.81 $[m/s^2]$

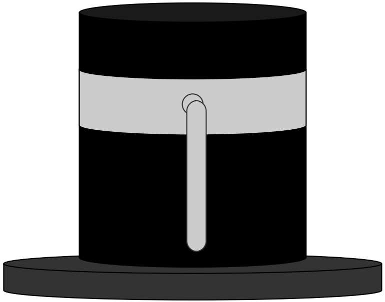


[1] Magnus Gäfvert, Modelling the Furuta Pendulum, ISSN 0280–5316

Modelling the Physical Phenomena[1]

State: $x = [\theta, \dot{\theta}, \dot{\phi}]^\mathsf{T}$, Input: u, Output: $y = x$

- θ is the pendulum angle (0 at the top position)
- $\dot{\theta}$ is the angular velocity
- $\dot{\phi}$ is the base velocity
- u is the torque applied at the base

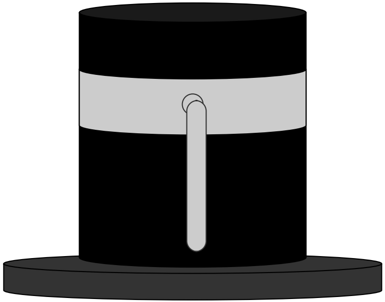


[1] Magnus Gäfvert, Modelling the Furuta Pendulum, ISSN 0280–5316

Modelling the Physical Phenomena[1]

Definitions

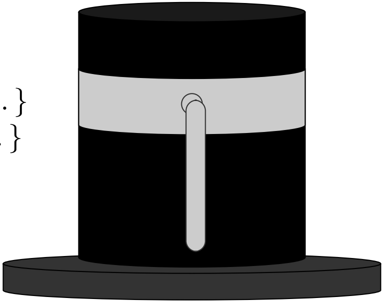
- $\alpha := J + m\,r^2 = 0.00013505856\,[kg\ m^2]$
- $\beta := {}^1\!/_3\,m\,l^2 = 0.0000102\,[kg\ m^2]$
- $\gamma := {}^1\!/_2\,m\,r\,l = 0.000008772\,[kg\ m^2]$
- $\delta := {}^1\!/_2\,m\,g\,l = 0.00200124\,[{}^{kg\ m^2}\!/_{s^2}]$



[1]Magnus Gäfvert, Modelling the Furuta Pendulum, ISSN 0280–5316

Modelling the Physical Phenomena[1]

$$\begin{cases} {}^{d\theta}/_{dt} & = & \dot{\theta} \\ {}^{d\dot{\theta}}/_{dt} & = & \left\{{}^{1}/_{\alpha\beta - \gamma^2 + (\beta^2+\gamma^2)\sin^2\theta}\right\} \left\{\beta\left(\alpha + \beta\sin^2\theta\right)\ldots\right\} \\ {}^{d\dot{\phi}}/_{dt} & = & \left\{{}^{1}/_{\alpha\beta - \gamma^2 + (\beta^2+\gamma^2)\sin^2\theta}\right\} \left\{\beta\,\gamma\left(\sin^2\theta - 1\right)\ldots\right\} \end{cases}$$

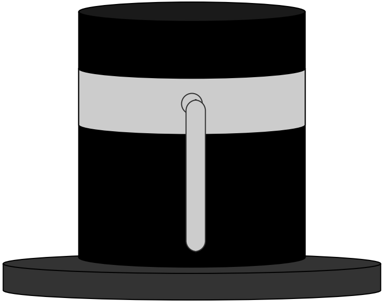


Non-linear model can be linearised around the equilibrium point in the top position, discretised – we use 0.005 $[s]$ as sampling period – and used for control design

[1] Magnus Gäfvert, Modelling the Furuta Pendulum, ISSN 0280–5316

Modelling the Physical Phenomena[1]

$$x_{k+1} = \underbrace{\begin{bmatrix} 1.0026 & 0.0050 & 0 \\ 1.0399 & 1.0026 & 0 \\ -0.0675 & -0.0002 & 1 \end{bmatrix}}_{A_d} x_k + \underbrace{\begin{bmatrix} -0.0843 \\ -33.7508 \\ 39.2131 \end{bmatrix}}_{B_d} u_k$$

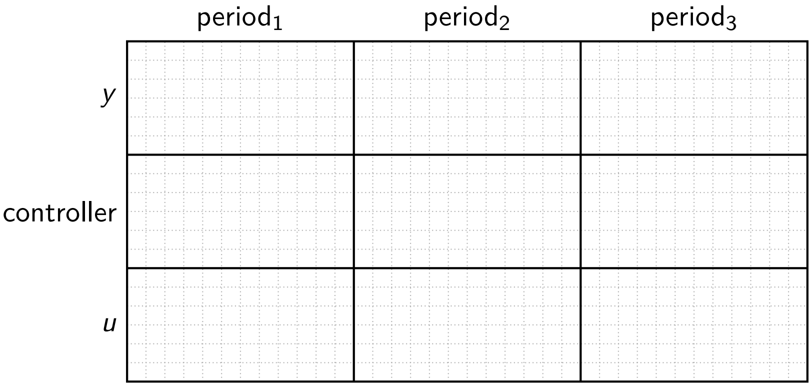


$$\lambda\left(A_d\right) = [1, 1.075, 0.93] \implies \text{unstable}^*$$

*: $\rho\left(A_d\right) = \max\left|\lambda\left(A_d\right)\right| = 1.075 > 1$

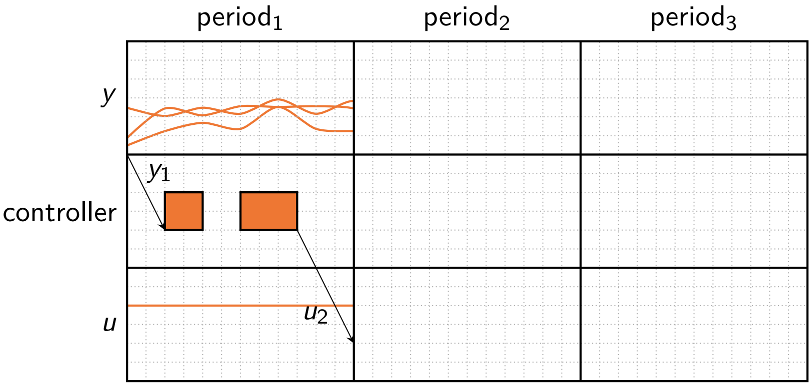
[1] Magnus Gäfvert, Modelling the Furuta Pendulum, ISSN 0280–5316

Designing the Control System[2]



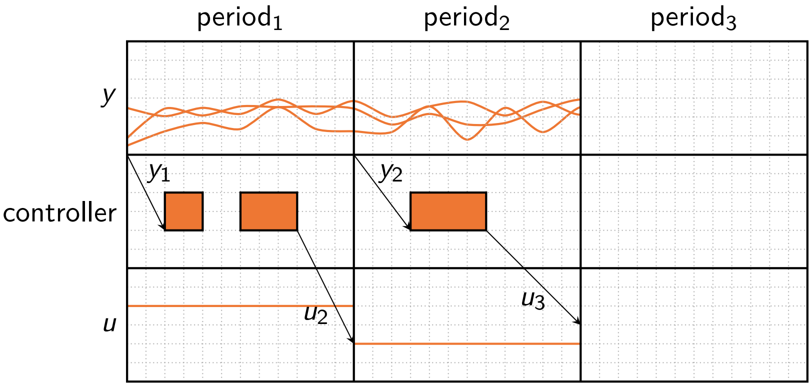
[2]Brindha Josephrexon and Martina Maggio, Experimenting with networked control software subject to faults, CDC 2022, `10.1109/CDC51059.2022.9992523`

Designing the Control System[2]



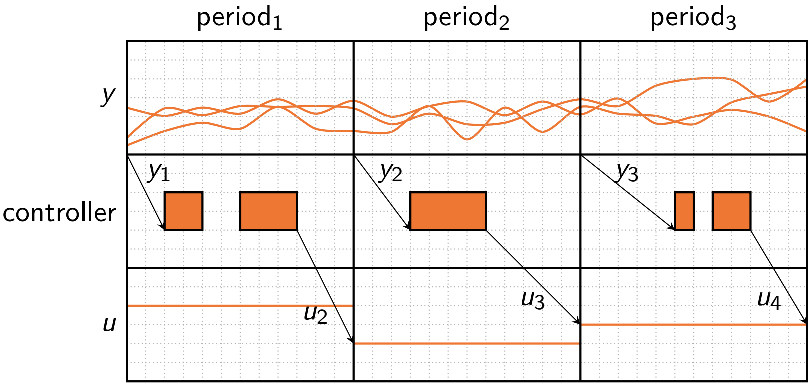
[2] Brindha Josephrexon and Martina Maggio, Experimenting with networked control software subject to faults, CDC 2022, `10.1109/CDC51059.2022.9992523`

Designing the Control System[2]



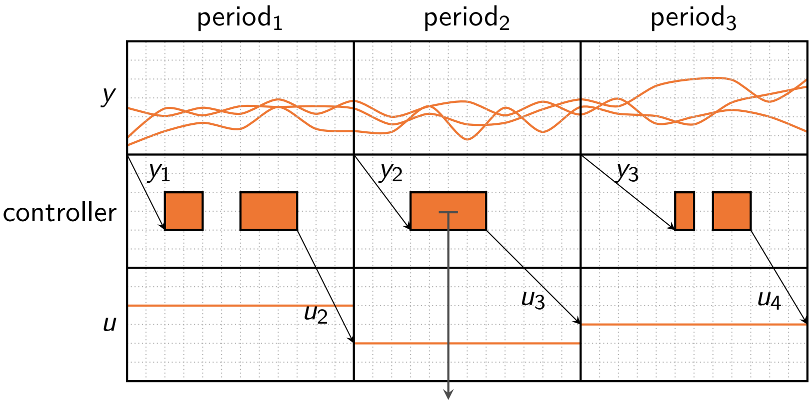
[2]Brindha Josephrexon and Martina Maggio, Experimenting with networked control software subject to faults, CDC 2022, `10.1109/CDC51059.2022.9992523`

Designing the Control System[2]



[2]Brindha Josephrexon and Martina Maggio, Experimenting with networked control software subject to faults, CDC 2022, `10.1109/CDC51059.2022.9992523`

Designing the Control System[2]



$$u_{k+1} = K\, y_k = K\, x_k = [0.375, 0.025, 0.0125]\, x_k$$

(One-Step Delay) Output (State) Feedback Controller

[2]Brindha Josephrexon and Martina Maggio, Experimenting with networked control software subject to faults, CDC 2022, `10.1109/CDC51059.2022.9992523`

Stability Verification (without deadline misses)

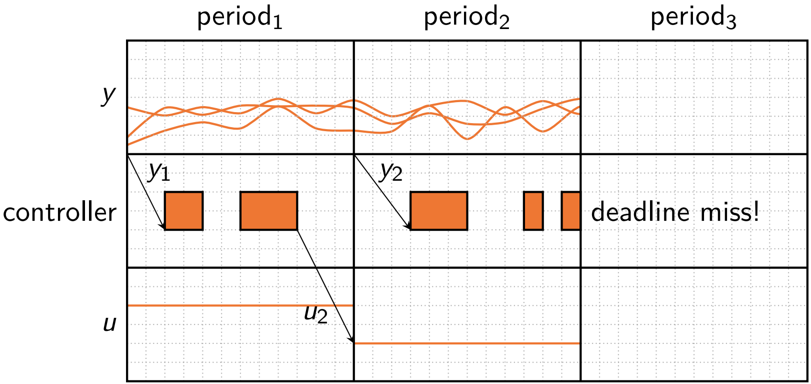
- Autonomous Closed-Loop System Behaviour: $x_{k+1} = A_d\, x_k + B_d\, K\, x_{k-1}$

$$\tilde{x}_k = \begin{bmatrix} x_k \\ x_{k-1} \end{bmatrix}, \; \tilde{x}_{k+1} = \begin{bmatrix} x_{k+1} \\ x_k \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & B_d\, K \\ I & 0 \end{bmatrix}}_{\Phi} \begin{bmatrix} x_k \\ x_{k-1} \end{bmatrix} = \Phi\, \tilde{x}_k$$

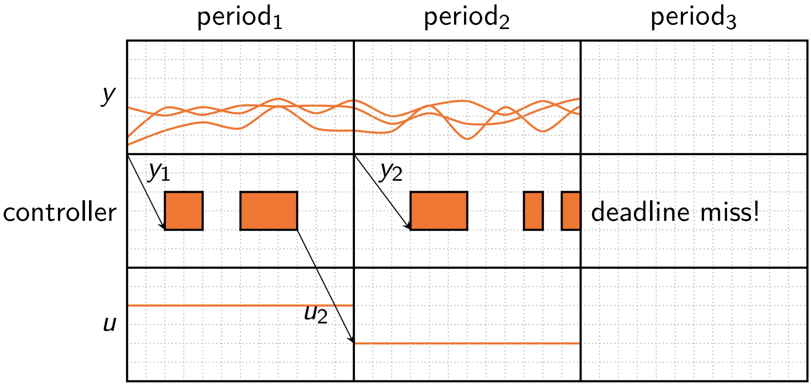
$$\lambda\left(\Phi\right) = [0.943, 0.727 \pm 0.181i, 0.609, 0, 0] \implies \text{stable}^*$$

*: $\rho\left(\Phi\right) = \max\left|\lambda\left(\Phi\right)\right| = 0.943 < 1$

What if there are deadline misses?



What if there are deadline misses?



2 choices: (i) value of u_3? (ii) execution in period$_3$?

What if there are deadline misses?

Control Signal[3]

- **Hold:** $u_{k+1} = u_k$ when a deadline is missed
- **Zero:** $u_{k+1} = 0$ when a deadline is missed

Task Execution[4]

- **Kill:** clean reset of the actions of the current job
- **Skip-Next:** continue the current job, don't start the next

[3] Luca Schenato, To Zero or to Hold Control Inputs With Lossy Links? IEEE Transactions on Automatic Control, 2009, 10.1109/TAC.2008.2010999; Steffen Linsenmayer and Frank Allgöwer, Stabilization of networked control systems with weakly hard real-time dropout description, CDC 2017

[4] Anton Cervin, Analysis of overrun strategies in periodic control tasks, IFAC World Congress 2005

Kill&Zero

Kill&Zero

Deadline Hit

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$
$$u_{k+1} = K\, x_k$$

Kill&Zero

Deadline Hit

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$
$$u_{k+1} = K\, x_k$$

$$\Downarrow$$

$$\begin{bmatrix} x_{k+1} \\ u_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & B_d \\ K & 0 \end{bmatrix}}_{\Phi_1} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

Kill&Zero

Deadline Hit

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$
$$u_{k+1} = K\, x_k$$
$$\Downarrow$$
$$\begin{bmatrix} x_{k+1} \\ u_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & B_d \\ K & 0 \end{bmatrix}}_{\Phi_1} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

Deadline Miss

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$
$$u_{k+1} = 0$$

Kill&Zero

Deadline Hit

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$
$$u_{k+1} = K\, x_k$$
$$\Downarrow$$
$$\begin{bmatrix} x_{k+1} \\ u_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & B_d \\ K & 0 \end{bmatrix}}_{\Phi_1} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

Deadline Miss

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$
$$u_{k+1} = 0$$
$$\Downarrow$$
$$\begin{bmatrix} x_{k+1} \\ u_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & B_d \\ 0 & 0 \end{bmatrix}}_{\Phi_0} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

Kill&Zero

Deadline Hit

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$
$$u_{k+1} = K\, x_k$$
$$\Downarrow$$
$$\begin{bmatrix} x_{k+1} \\ u_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & B_d \\ K & 0 \end{bmatrix}}_{\Phi_1} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

The closed-loop system switches arbitrarily between Φ_1 and Φ_0

Deadline Miss

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$
$$u_{k+1} = 0$$
$$\Downarrow$$
$$\begin{bmatrix} x_{k+1} \\ u_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & B_d \\ 0 & 0 \end{bmatrix}}_{\Phi_0} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

Kill&Zero

Deadline Hit

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$
$$u_{k+1} = K\, x_k$$

$$\Downarrow$$

$$\begin{bmatrix} x_{k+1} \\ u_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & B_d \\ K & 0 \end{bmatrix}}_{\Phi_1} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

The closed-loop system switches arbitrarily between Φ_1 and Φ_0

Deadline Miss

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$
$$u_{k+1} = 0$$

$$\Downarrow$$

$$\begin{bmatrix} x_{k+1} \\ u_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & B_d \\ 0 & 0 \end{bmatrix}}_{\Phi_0} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

We don't have much hope to guarantee stability...

Kill&Zero

Deadline Hit

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$
$$u_{k+1} = K\, x_k$$
$$\Downarrow$$
$$\begin{bmatrix} x_{k+1} \\ u_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & B_d \\ K & 0 \end{bmatrix}}_{\Phi_1} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

The closed-loop system switches arbitrarily between Φ_1 and Φ_0

Deadline Miss

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$
$$u_{k+1} = 0$$
$$\Downarrow$$
$$\begin{bmatrix} x_{k+1} \\ u_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & B_d \\ 0 & 0 \end{bmatrix}}_{\Phi_0} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

We don't have much hope to guarantee stability...
...unless we constrain the switches!

Constraint #1: Consecutive deadline misses[7]

- **Constraint:** We cannot miss more than n consecutive deadlines
- **Analysis:** Calculating upper bounds[5] of the joint spectral radius[6] of the set

$$\Sigma = \{\Phi_1 \, \Phi_0^i \,|\, i \in \mathbb{Z} \wedge 0 \leq i \leq n\}$$

[5] Vincent Blondel and John Tsitsiklis, The boundedness of all products of a pair of matrices is undecidable, Systems & Control Letters, 2000; Guillaume Vankeerberghen, Julien Hendrickx, and Raphaël M. Jungers, JSR: a toolbox to compute the joint spectral radius, HSCC 2014

[6] Gian-Carlo Rota and Gilbert Strang, A note on the joint spectral radius Indagationes Mathematicae 1960; Raphael Junger, The Joint Spectral Radius: Theory and Applications, Lecture Notes in Control and Information Sciences, 2009

[7] Martina Maggio, Arne Hamann, Eckart Mayer-John, Dirk Ziegenbein, Control System Stability under Consecutive Deadline Misses Constraints, ECRTS 2020

Constraint #2: Weakly-Hard Constraints

- **Constraint:** The number of deadline hits/misses in a window of consecutive execution is constrained by the weakly-hard[8] task model (sequences of 0s and 1s)
- **Analysis:** (i) building the finite state machine[9] that recognises the language defined by a weakly-hard constraint, (ii) use the directed adjacency matrix of the finite state machine, the Kronecker lifting and the joint spectral radius[10]

[8] Guillem Bernat, Alan Burns, Albert Liamosí, Weakly hard real-time systems, IEEE Transactions on Computers 2001

[9] Nils Vreman, Richard Pates, and Martina Maggio, WeaklyHard.jl: Scalable Analysis of Weakly-Hard Constraints, RTAS 2022, `https://github.com/NilsVreman/WeaklyHard.jl`

[10] Nils Vreman, Paolo Pazzaglia, Victor Magron, Jie Wang, Martina Maggio, Stability of Linear Systems Under Extended Weakly-Hard Constraints, CDC & Control Systems Letters 2022

Constraint #3: Probabilistic[12]

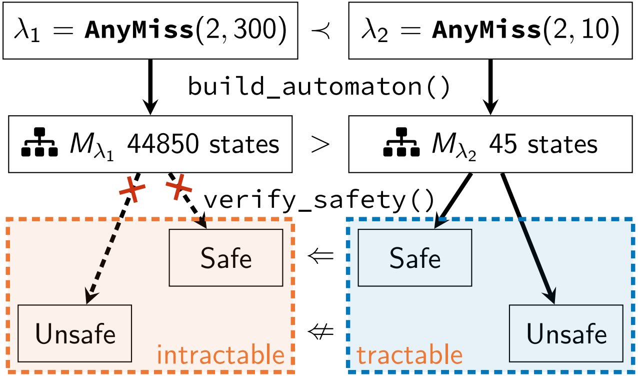
- **Constraint:** The control task misses deadlines with probability p_c (additionally, sensor messages are not delivered with probability p_s and actuator messages are not delivered with probability p_a)
- **Analysis:** using Markov Jump Linear Systems theory and proving mean square stability,[11] implying almost sure convergence

[11] Y. Fang and K.A. Loparo and X. Feng, Almost sure and δ-moment stability of jump linear systems, International Journal of Control 1994

[12] Nils Vreman and Martina Maggio, Stochastic Analysis of Control Systems Subject to Communication and Computation Faults, EMSOFT 2023

Open Problem

- Desire to verify stability with constraints such as **AnyMiss**$(2, 300)$
- The constraint needs to be approximated with a more conservative one (i.e., one that has a higher number of deadline misses)
- Safety of the conservative constraint implies safety of the non-conservative one



Conclusion

- Sometimes it is possible to guarantee stability even when deadlines are missed providing that the irregularities follow some prescribed pattern
- Thanks to many collaborators: Arne Hamann, Brindha Jeniefer Josephrexon, Victor Magron, Claudio Mandrioli, Eckart Mayer-John, Richard Pates, Paolo Pazzaglia, Nils Vreman, Jie Wang, Dirk Ziegenbein
- Questions? `maggio@cs.uni-saarland.de`